

Kinetic model of a plasma in contact with an ion-emitting anode surface

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Abstract: A collision-less, one-dimensional kinetic model was developed to describe the potential profile within a plasma sheath in contact with an anode surface where the anode emits ions into the plasma. Solutions are presented in normalized form, describing a wide range of emitted ion currents and bulk plasma parameters. The neutralization parameter was found to not directly correlate to plasma neutrality. Constraints were given for the potential drop across the sheath that are required for monotonically increasing potential profiles. It was found that as the emitted current from the wall increased, the potential drop across the sheath decreased in magnitude. Also, for the same plasma conditions and potential drop across the sheath, multiple potential profiles existed.

Nomenclature

α	= Neutralization Parameter $\left(\frac{qN_{si}}{eN_{se}}\right)$
β_e	= $\frac{m_e}{2kT_{se}}$
β_i	= $\frac{m_i}{2kT_{si}}$
γ	= Net current through sheath, normalized by electron thermal current
$\delta()$	= Impulse function
ϵ_0	= Permittivity of free space
$\theta()$	= Heaviside step function
λ	= Current of ions from wall, normalized by electron thermal current
$\rho(\varphi, \varphi_{wall})$	= Charge density at a given position in the sheath for a give wall potential
$\varphi(x)$	= Electrostatic potential in the sheath
φ_{wall}	= Electrostatic potential between the wall and plasma
$\psi(\varphi)$	= Potential in sheath above plasma potential, normalized by ion temperature
$\psi_e(\varphi)$	= Potential in sheath above plasma potential, normalized by electron temperature
$\psi_i(\varphi)$	= Difference in potential between ψ_w and $\psi(\varphi)$
ψ_w	= Wall potential in sheath above or below plasma potential, normalized by ion or electron temperature
e	= Elementary charge on an electron
$\text{erfc}()$	= Complementary error function
E_{wall}	= Electric field at the wall (assumes electric field at plasma/wall boundary is zero)
F_e	= Velocity distribution function for electrons
\vec{F}_e	= Electron flux
F_i	= Velocity distribution function for ions
\vec{F}_i	= Ion flux

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\bar{F}_{ic}	=	Emitted ion flux
J_e	=	Normalized electron current
k	=	Boltzmann constant
M_e	=	Electron Mass
M_i	=	Ion Mass
$N_e(\varphi)$	=	Density of electrons at a given position in the sheath
$N_i(\varphi)$	=	Density of ions at a given position in the sheath
N_{se}	=	Density of electrons entering the sheath from the Maxwellian plasma
N_{si}	=	Density of ions entering the sheath from the Maxwellian plasma
q	=	Elementary charge of an ion
T_{se}	=	Temperature of electrons in Maxwellian plasma
T_{si}	=	Temperature of ions in Maxwellian plasma
τ	=	Temperature Ratio of ions to electrons in the Maxwellian plasma $\left(\frac{T_{si}}{T_{se}}\right)$
V_{me}	=	Minimum velocity of electrons due to electrostatic potential
V_{mi}	=	Minimum velocity of ions due to electrostatic potential
v_x, v_y, v_z	=	Particle velocity in the x, y and z (respectively) directions
$x(\varphi)$	=	Distance from sheath/plasma boundary, normalized by Debye length

I. Introduction

THE non-neutral sheath that forms between an equilibrium plasma and a solid surface has been the subject of countless studies since the beginning of plasma physics.^[1-5] Kinetic models have been developed that predict the potential profile within the sheath as a function of the densities and temperatures within the bulk plasma.^[4, 6-8] Under certain circumstances the boundary wall, may emit electrons. This can happen, for instance, in the case of secondary electron emission due to particle impacts, field-emission from micro-protrusions on a cathode surface, or thermionic emission from a heated wall. For these situations, kinetic sheath models that account for current emission from a wall have been developed to describe the change in potential structure caused by the emitted flux of charge.^[8-10] McIntyre's model^[9] parametrically analyzed different families of potential profiles. Ordoñez^[8] created a fully kinetic sheath model that predicted the potential profiles for a wall that was capable of electron emission. Riemann^[11] created a kinetic-hydraulic hybrid model that studied sheath formation and Bohm criterion when a fraction of the ions were able to reflect off of the cathode wall and hot ions could be emitted off a cathode wall. Riemann's model was primarily interested in the Bohm criterion when a cathode could reflect ions off of its surface. His model did not model the case of an anode capable of ion emission. To date, none of the kinetic models available model the effect of emitted ions from an anode wall; however, there are certain plasma cases where the wall may be capable of emitting ions.

In the fusion processes described in reference [12] the objective was to use a strong electric field to extract Li ions directly from a liquid-lithium-coated anode. These lithium ions were extracted due to a strong electric field at the surface of the liquid metal. In reference [12], the electric field was formed in vacuum by biasing a cathode plate downstream of the liquid-coated anode in a process known as electrospray. The field required to induce electrospray depends on the surface tension of the stressed liquid. In the case of liquid metals such as lithium, this electric field must be on the order of 10^7 V/m^[13] in order to overcome the tension. However, room-temperature molten salts known as ionic liquids have conductivities similar to liquid metals, but with surface tensions an order of magnitude lower. The work reported here is part of a project at Michigan Technological University to determine whether the electric field that forms naturally within a sheath separating a gaseous plasma from the surface of an ionic-liquid-coated anode can induce electrospray ion emission from the liquid. In both the lithium and the ionic liquid cases, emitted ion current is caused by the electric field at the liquid surface. In the lithium vacuum-electrode case this field was created by a downstream cathode while in the ionic-liquid plasma interface this field is created by the charge imbalance in the plasma sheath. However, in both cases the emitted ion current will subsequently modify the surface electric field due to space charge and, thus, the potential structure and the surface emission are coupled.

Charge injection from the wall will weaken the electric field within the sheath. If the sheath electric field is responsible for the charge emission (e.g. sheath field at the anode surface pulls ions from the liquid) then the interaction between the emitted current and the sheath potential must be studied self-consistently to determine the range of possible emission current. While the phenomenology of a cathode surface emitting electrons is exactly the same as an anode surface emitting ions it is expected that the sheath profiles of emitting anodes will be different than the potential profiles of emitting cathodes because ions are heavier and thus less mobile. This may limit the

current of extracted ions due to space charging within the sheath. The goal of this paper was to develop a sheath model that could describe a sheath in contact with a wall that is capable of ion emission.

II. Derivation of Kinetic Equations

The model described in this paper is a kinetic model of the sheath that forms between a Maxwellian plasma and a planar surface that has the ability to emit ions. For the purposes of this study the wall is assumed to be ‘ion repelling,’ that is the wall potential is higher than that of the Maxwellian plasma. Such a situation would be expected to occur at the surface of an anode within a plasma discharge in the case of a positive anode fall. A schematic of the sheath model is given in Figure 1. At the plasma-sheath boundary, ions and electrons are allowed to enter the sheath from the Maxwellian plasma. The ions that enter the sheath, but have insufficient energy to reach the wall, are refluxed back towards the plasma-sheath boundary. The boundary of the wall absorbs and/or neutralizes the ions and electrons that reach it from the plasma. The wall is also a source for emitting cold ions into the sheath at a constant current.

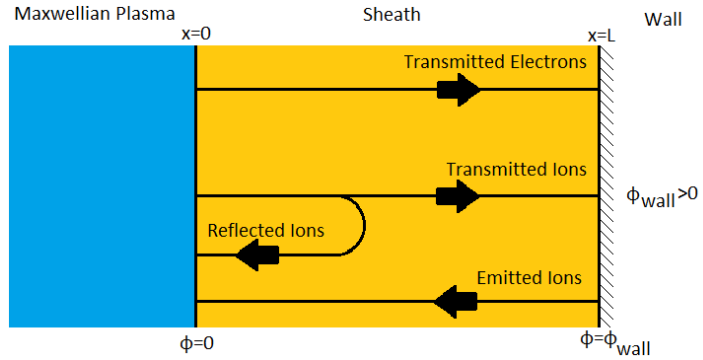


Figure 1. Schematic of plasma-sheath.

In this model there are four different groups of particles whose velocity distributions must be considered. The first is electrons that originate in the Maxwellian plasma, enter the sheath, and are accelerated until they reach the wall and are absorbed/neutralized. The second group is singularly charged ions that originate from the Maxwellian plasma, decelerate in the sheath, and eventually reach the wall and are absorbed/neutralized. The third is ions that were part of the second group, but have insufficient energy to reach the the wall and are reflected back toward the plasma-sheath boundary. The last group is the ions that originate at the wall, travel through the sheath due to the electrostatic potential and end in the plasma.

A number of qualitatively different potential profiles are possible in the sheath separating a plasma from an emitting wall. An excellent summary of these profiles was provided by Ott^[4], with a key figure reproduced here as Figure 2. In Figure 2 the neutralization parameter, α , is plotted on the y-axis with the normalized wall potential, η_c plotted on the x-axis. The neutralization parameter, α , is defined as the ratio of ions to electrons that originate in the plasma (the emitter surface in Ott's model) that also have a positive velocity. The normalized wall potential, η_c , is the potential difference between the plasma and the wall normalized by temperature (in Ott's model, the ions and electrons had the same temperature). In Ott's model, electrons and ions were created at an emitter with a given temperature. This emitter was conceptually equivalent to the bulk Maxwellian plasma since it served as the source of the Maxwellian ions and electrons. Ott also had a collecting wall which was biased with respect to the emitter. When the collector was biased positively with respect to the emitter, the collector acted as an anode and all results with this configuration map to the right half plane of Figure 2. All results in the left half plane of Figure 2 are when the collector was biased negative with respect to the emitter, acting as a cathode wall. Ott showed that the α - η_c space can be subdivided into different regions defined by the general shape of the potential profile. For instance, all sheath profiles within region 1A (between the curves a and c) demonstrated a single, monotonically increasing sheath while profiles between curves c and d exhibit a single minima and have an electric field which goes to zero at some point within in sheath. Potential profiles can be calculated parametrically anywhere in the α - η_c space, however one or more of the parameters must be constrained in order to close the problem for a particular case. Ordoñez chose to limit his model to the case

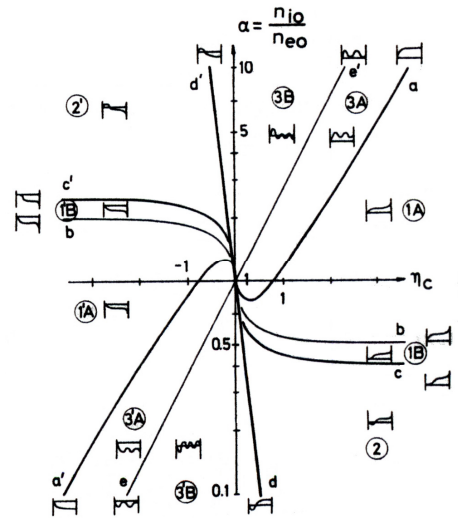


Figure 2. Regions of various potential distributions in parameter space, from Ott.

where the electric field was (1) zero at the plasma/sheath boundary, and (2) zero at some intermediate point within the sheath. By doing this, the Ordoñez model was able to constrain the neutralization parameter, α , and obtain a closed solution for a given set of plasma conditions, however this approach limited the sheaths to those that would exist along the line c' in Ott's sheath space (Figure 2).

The model derived here is a modification of Ordoñez's electron emitting wall model^[8] with the boundary conditions modified to allow for a wall to emit ions instead of electrons. The presented model has three key differences from the Ordoñez model. First, the wall is able to emit ions and thus must be able to repulse them, meaning the wall potential must be greater than zero. So, while Ordoñez's model was concerned with the left-half plane of Ott's space the present model addresses the right-half plane. Second, the present model does not require the electric field to go to zero at a point within the sheath. In the present model, α is allowed to be defined by the user, so long as it fits in the realm of monotonic potential profiles, which encompasses all possible potential profiles in region 1A and 1B, including lines a, b and c in Figure 2. Third, the emitted current from the wall is not related to any incident flux. For an electron-emitting wall the emitted current is often due to secondary electron emission, where the impact of an incident electron causes ejection of an electron from the wall. Thus, the emitted current is conveniently expressed as a yield coefficient, which is some fraction of the incident electron flux. For the ion-emitting wall case described here the ion emission flux will likely be due to the electric field at the wall rather than the flux of impacting particles, so the definition of a yield coefficient is not relevant.

A. Assumptions of model

The present model is a fully kinetic sheath model. It assumes that the sheath is collision-less. The plasma outside of the sheath is Maxwellian and has only a single species of ions. The ions that are emitted from the wall are "cold" (emitted with zero velocity), and are only accelerated because of the electrostatic potential due to the charge imbalance. Also, the particle motion is restricted to the x -axis. Another assumption in this model was that the ion species in the plasma is the same species that is emitted from the wall, and all ions are singularly charged. This model is valid for a fully developed, steady state system. The electrostatic potential is required to be monotonically increasing from the plasma to the wall, such as would be found in a positive anode fall sheath. This model does not account for any other forces such as a transverse magnetic field impeding the motion of the electrons. The current polarity is defined such that ion flux from the plasma to the wall denotes a positive current through the sheath. The electric field and electrostatic potential were defined to be zero at the Maxwellian plasma-sheath boundary.

B. Definition of terms

The nomenclature is chosen to be equivalent to that used by Ordoñez where possible. The normalized potentials are given as $\psi_e = \frac{e\phi}{kT_{se}}$, $\psi = \frac{q\phi}{kT_{si}}$, $\psi_w = \frac{q\phi_{wall}}{kT_{si}}$, and $\psi_i = \psi_w - \psi$. The minimum velocity that an electron will have at a given sheath potential is defined as $V_{me} = \sqrt{\frac{\psi_e}{\beta_e}}$, and for an ion $V_{mi} = \sqrt{\frac{\psi_i}{\beta_i}}$. The neutralization parameter, $\alpha = \frac{qN_{si}}{eN_{se}}$, is the ratio of charge densities of ions to electrons in the Maxwellian plasma.

C. Electron velocity distribution equations

The electron velocity distribution function, Eq.(1), is based on the solution to the Vlasov equation from Ordoñez^[8]. The argument of the Heaviside step function in Eq.(1) sets the minimum value of the integrals when taking the moments. This accounts for the velocity increase of the electrons due to the electrostatic potential that accelerates them.

$$F_e = N_{se} \left(\frac{\beta_e}{\pi}\right)^{3/2} e^{(-\beta_e v^2 + \psi_e)} \theta(v_x - V_{me}) \quad (1)$$

The zeroth moment of the electron velocity distribution function yields the particle density, shown in Eq.(2).

$$N_e = \iiint_{-\infty}^{\infty} F_e d^3v = \frac{1}{2} N_{se} e^{\psi_e} \operatorname{erfc}(\sqrt{\psi_e}) \quad (2)$$

The electron flux is the first moment of the electron velocity distribution function.

$$\vec{\mathcal{F}}_e = \iiint_{-\infty}^{\infty} v F_e d^3v = \left[\frac{N_{se}}{2\sqrt{\pi}\beta_e} \right] \hat{\mathbf{t}} \quad (3)$$

D. Ion velocity distribution equations

The ion velocity distribution function is comprised of three parts; transmitted ions (F_i^t), reflected ions (F_i^r), and emitted ions (F_i^e). The transmitted ions originate in the plasma and have enough energy to impact the wall. The reflected ions also originate from the plasma, but their energy is insufficient to reach the wall and they are returned to the plasma. The emitted ions are born on the wall and end in the plasma.

$$F_i = F_i^t + F_i^r + F_i^e \quad (4)$$

1. Transmitted Ion velocity distribution function

The transmitted ion distribution function includes all ions with a positive x-velocity. This distribution only accounts for ions that have enough energy to keep moving towards the wall. The ions in this distribution may or may not have enough energy to reach the wall.

$$F_i^t = N_{si} \left(\frac{\beta_i}{\pi}\right)^{3/2} e^{(-\beta_i v^2 - \psi)} \theta(v_x) \quad (5)$$

2. Reflected Ion velocity distribution function

The reflected ion distribution function includes all ions with a velocity between $-V_{mi}$ (ions almost reaching the wall but were turned around) and a velocity of zero (the ions that are stopped at that given potential).

$$F_i^r = N_{si} \left(\frac{\beta_i}{\pi}\right)^{3/2} e^{-\beta_i v^2 - \psi} \theta(-v_x) \theta(v_x + V_{mi}) \quad (6)$$

3. Emitted Ion velocity distribution function

The velocity of all of the emitted ions is governed only by the electrostatic potential, or equal to $-V_{mi}(\phi)$ at each point in the sheath. It also assumes that the emitted ions are emitted with no y- or z-velocity component. The velocity distribution function for the emitted ions is defined as the flux of the emitted ions divided by the average of the velocity of the emitted ions.

$$F_i^e = - \left(\frac{\bar{\mathcal{F}}_{ic}}{\langle v_x \rangle}\right) \delta(v_x + V_{mi}) \delta(v_y) \delta(v_z) \quad (7)$$

4. Ion Density and Flux

The zeroth moment of the ion velocity distribution function yields the ion particle density. The ion particle density used the combined ion distribution functions from the three separate velocity distributions.

$$N_i = \iiint_{-\infty}^{\infty} F_i d^3v = \frac{N_{si}}{2} e^{-\psi} \operatorname{erfc}(-\sqrt{\psi_i}) + \frac{\bar{\mathcal{F}}_{ic}}{V_{mi}} \quad (8)$$

Just like Eq. (8), in Eq. (9) the flux of emitted ions, $\bar{\mathcal{F}}_{ic}$, is temporarily left undefined. It will be defined later, in terms of the normalized emitted current, λ .

$$\bar{\mathcal{F}}_i = \iiint_{-\infty}^{\infty} v F_i d^3v = \left[\frac{N_{si} e^{-\psi_w}}{2\sqrt{\pi\beta_i}} \right] \hat{i} + \bar{\mathcal{F}}_{ic} \quad (9)$$

E. Solving Poisson's Equation

The net current through the sheath, γ , is normalized by the electron thermal current. The three terms in Eq.(10) are the ion current from the plasma, the emitted ion current, and the electron current, respectively.

$$\gamma = \frac{q\bar{\mathcal{F}}_i - e\bar{\mathcal{F}}_e}{\frac{eN_{se}}{2\sqrt{\pi\beta_e}}} = \alpha \sqrt{\frac{\beta_e}{\beta_i}} e^{-\psi_w} - \frac{q\bar{\mathcal{F}}_{ic}}{2\sqrt{\pi\beta_e}} - 1 \quad (10)$$

In Eq. (11), λ replaces the terms in Eq.(10) which represent the emitted ion current. For reference, the electron current is defined as J_e and is equal to the '-1' value in Eq. (11).

$$\gamma = \alpha \sqrt{\frac{\beta_e}{\beta_i}} e^{-\psi_w} - \lambda - 1 \quad (11)$$

The emitted ion flux is the emitted ion current, divided by charge multiplied by the normalization factor of thermal electron current.

$$\bar{\mathcal{F}}_{ic} = \lambda \frac{eN_{se}}{2q\sqrt{\pi\beta_e}} \quad (12)$$

Rearranging Eq. (11) and solving for the wall potential yields Eq. (13). The wall potential is determined based on the values chosen for the other parameters; the plasma conditions, the net current and current emitted from the wall.

$$\varphi_{wall} = -\frac{kT_{si}}{q} \ln \left(\frac{1+\lambda+\gamma}{\alpha} \sqrt{\frac{\beta_i}{\beta_e}} \right) \quad (13)$$

The parameter φ_{wall} must be real and positive. To satisfy this condition, the argument of the natural logarithm in Eq.(13) must be between zero and one giving the limiting condition in Eq.(14). The numerical value of the emitted current, λ , is either zero or a positive value. The negative sign in front of the λ term in Eq. (11) accounts for the direction the ions are traveling with respect to current polarity as defined in Section II A.

$$-1 - \gamma \leq \lambda \leq \alpha \sqrt{\frac{\beta_e}{\beta_i}} - 1 - \gamma \quad (14)$$

The charge density at any given point is given in Eq.(15).

$$\rho = qN_i - eN_e \quad (15)$$

A version of Poisson's equation is given in Eq.(16) which can be rearranged and integrated to arrive at Eq.(17). Equation (17) can be numerically integrated from the bulk plasma potential, which is defined to be zero, up to the wall potential, φ_{wall} , giving the function $x = x(\varphi)$, which can then be numerically inverted to obtain the potential structure within the sheath, $\varphi=\varphi(x)$.

$$\frac{d}{dx} \left(-\frac{d\varphi}{dx} \right)^2 = -\frac{2\rho}{\epsilon_0} \frac{d\varphi}{dx} \quad (16)$$

$$x(\varphi) = -\int_0^\varphi \left(\frac{-2}{\epsilon_0} \int_0^\varphi \rho d\varphi \right)^{-1/2} d\varphi \quad (17)$$

In order to maintain a real and finite position for each sheath potential, the argument of the square root must be positive and non-zero. To check this, the integral of the charge density can be calculated using Eq.(18). This means the total charge between the plasma-sheath boundary and a given potential must always be negative. For this reason, the charge density at a given sheath potential could be zero and/or positive; however, the integral of the charge densities between the plasma-sheath boundary and the potential must remain negative.

$$I = \int_0^\varphi \rho d\varphi \quad (18)$$

III. Results

All of the results presented in this paper are generalized. All of the potentials, currents and distances are normalized to plasma parameters such as the temperature ratio of ions to electrons. For all of the results presented, the constant plasma parameters are equal ion and electron temperature ($\tau = 1$), the selected ion species was chosen to have an atomic mass of 131 amu (singularly charged Xenon).

A. Discussion of the neutralization parameter, α

Generally, Maxwellian plasmas are neutral (or at least quasi-neutral). In the model described in this paper, the neutralization parameter, α , is defined as the ratio of ion density to electron density that enters the sheath from the bulk plasma. Upon first inspection, one would expect the neutralization parameter, α , to be close to one for a quasi-neutral plasma. If a condition of requiring the electric field to go to zero at a point within the sheath is enforced, such as in Ordoñez's model^[8], the solutions are mapped to lines c and c' of Figure 2. Depending on wall potential, the neutralization parameter appears to vary between $\alpha=0.4$ and $\alpha = 2.5$. This would seem to imply that the bulk plasma is far from neutral.

An explanation of the range of the neutralization parameter, α , is best conveyed when a 1-D system with a cathode and an anode, like that presented in Figure 3, is examined. In a cathode sheath, all ions that enter the sheath are transmitted through the sheath and are absorbed or neutralized at the cathode wall. A portion of the electrons that enter the cathode sheath reach the cathode wall and are absorbed or neutralized, but a fraction of those that enter the sheath from the plasma are reflected back into the plasma, with a Maxwellian energy distribution. These reflected electrons have a flux in the direction of the anode sheath, thus increasing the number of electrons that reach the anode sheath. The ratio of ions to electrons that reach the anode sheath edge is the ratio of the density of the ions to the electrons in the quasi-neutral Maxwellian plasma, divided by the complimentary error function, as denoted in the lower right of Figure 3. The erfc ranges from 1 to 2. Therefore, the neutralization parameter can vary from

unity and can be as small as 0.5 even for a quasi-neutral bulk plasma. A similar analysis can be applied to the cathode. Since both the anode sheath and cathode sheath are ejecting charged particles, the Maxwellian plasma can maintain quasi-neutrality.

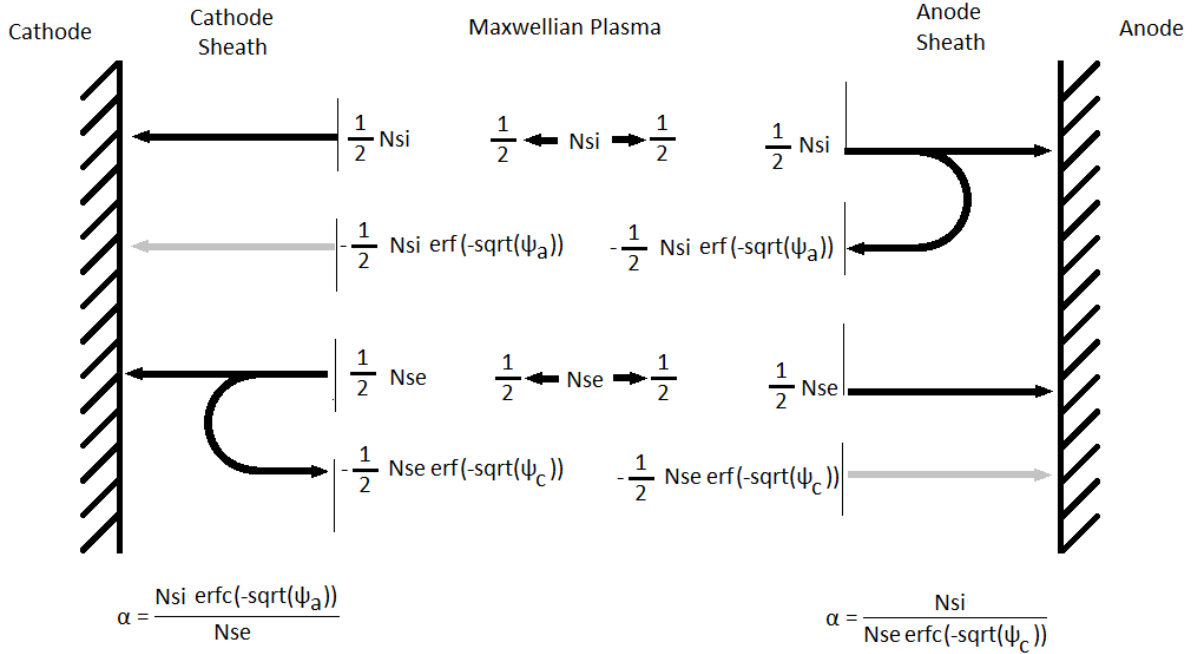


Figure 3. One Dimensional Sketch of a plasma with anode and cathode boundaries. The potential drop across the cathode sheath is ψ_c and ψ_a is the potential drop across the anode sheath.

Unlike the Ordoñez model^[8], the model described in this paper did not restrict the solution space to only those profiles where the electric field is zero at some point within the sheath. Because of this, the neutralization parameter, α , becomes a free parameter. In addition, there is no way to have a closed solution, but instead a set of solutions. A closed solution would imply a single independent variable exists. A solution set is required because there are multiple independent variables: α , γ , τ , and λ .

B. Changes in potential profiles due to total and emitted currents

The potential profiles in and potential drop across an anode sheath can vary because of changes in the bulk plasma, the current conducted through the sheath or the amount of current emitted from the anode surface. In this section, the potential profiles in the sheath will be examined as the total current through the sheath, γ , and emitted current, λ , are varied, but the bulk plasma is maintained constant.

Each of the curves on Figure 4(a) and Figure 4(b) represent the potential profile in the sheath for a given emitted current. For these graphs, the neutralization parameter, α , is held constant. This condition allows for the comparison of various sheaths while maintaining constant plasma conditions. In Figure 4(a), the $x=0$ location is the plasma-sheath boundary, and the potential at this location for all profiles is zero. The magnitude of the potential in each curve increases until it reaches the wall, which is where each curve ends. This point occurs at different values of x for each profile since the sheath thicknesses are different. Since each potential profile in Figure 4(a) has the same net current, γ , and electron current, J_e , when the emitted ion current, λ , varies, the current of the ions from the plasma changes one-to-one with it, ($\alpha \sqrt{\frac{\beta_i}{\beta_e}} e^{-\psi_w} - \lambda = const$). The transmitted ion term in Eq.(11), $\alpha \sqrt{\frac{\beta_i}{\beta_e}} e^{-\psi_w}$, is the normalized ion current from the plasma. From this, in order to increase the transmitted ion current, the wall potential must drop. When the wall potential decreases in magnitude, it allows more of the ions from the plasma to reach the wall instead of being returned to the plasma. Because of this, as the emitted current, λ , increases, the magnitude of potential drop across the sheath decreases.

When the total current through the sheath, γ , is varied, the allowable ranges of transmitted ion current and emitted ion current changes. If no ions are emitted from the wall, as the magnitude of the total current through the sheath, γ , increases, the number of ions reaching the wall must decrease, meaning a higher wall potential. In both Figure 4 (a) and (b), the blue curve denotes the potential profile with the no ions emitted from the wall. The

magnitude of total current through the sheath, γ , for the potential profiles in Figure 4(b) is greater than that of the potential profiles in Figure 4(a). The wall potentials in Figure 4(b), are larger than those in Figure 4(a), because of the difference in total current through the sheath. Another observation is as the magnitude of total current through the sheath increases, so does the length of the sheath. One possible explanation for this is as the wall potential increases, the ion density at the plasma-sheath boundary increases due to an increase in reflected ions. Because of the higher ion density at the boundary, the ratio of ions to electrons becomes closer to unity, therefore the electric field at that point is not as high, and thus the potential increases at a lower rate.

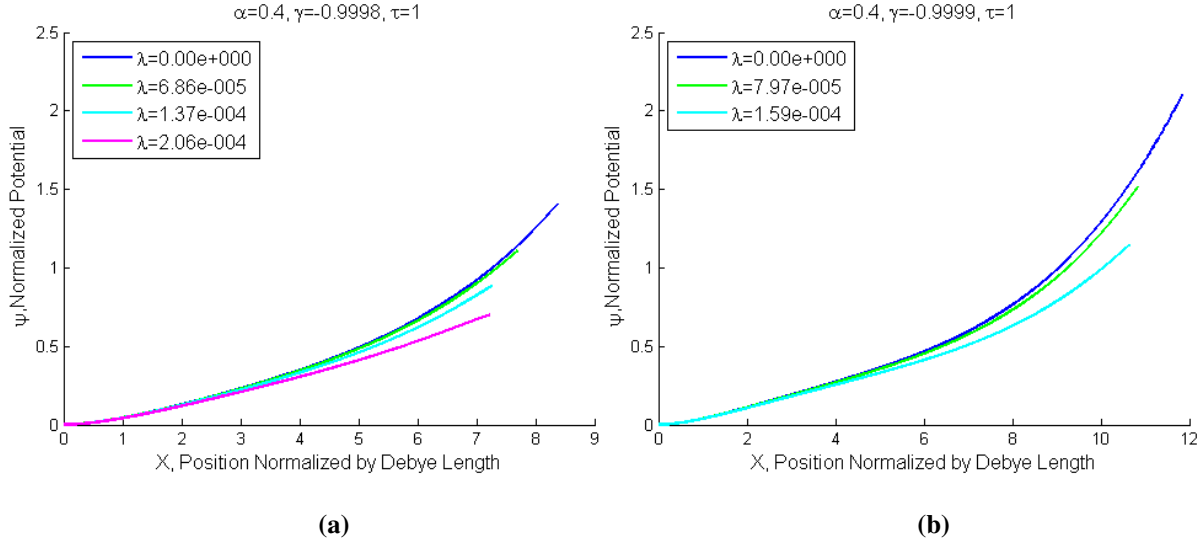


Figure 4. Potential profile of ion emitting wall with $\tau = 1$, various λ and (a) $\gamma = -0.9998$ (b) $\gamma = -0.9999$.

The normalized total current through the sheath, γ , has a range that is very close to -1. This is in part because the total current is normalized by the electron thermal current. The electron current, J_e , is equal to -1 because all of the electrons that reach the plasma-sheath boundary are accelerated towards the anode wall. The current from the ions that originate in the bulk plasma is much smaller than the current due to the electrons from the plasma. First, because the ions are much more massive than the electrons, they travel much slower and their current is much lower. Second, not all of the ions that enter the sheath from the plasma are able to reach the anode wall and contribute to the current.

The range of current due to ions that originate in the plasma is limited by their mass ratio to electrons and the normalized wall potential, ψ_w . In the limit of zero wall potential, all of the ions that enter the sheath from the plasma reach the anode wall and is the coincides with maximum current from the ions from the plasma. The maximum current from the ions from the plasma is much lower than electron current, J_e . For instance, if the ions are six orders of magnitude heavier, the maximum current from the ions is three orders of magnitude lower than the electron current. Because of this, for small changes in total current through the sheath, γ , the wall potential, ψ_w , can change drastically.

C. Analysis of solutions in α - ψ_w parameter space

Instead of defining the neutralization parameter, α , a new term was defined; the ratio of ion to electron densities at the $\phi = 0$ point, or $\frac{N_i(0)}{N_e(0)}$. This was done as a way to try and understand how the neutralization parameter, α , changed with respect to wall potential, ψ_w , and if the neutralization parameter was related back to the neutrality of the bulk plasma. The former is described below while the later was described above in Section III A.

For each marker type (circle and asterisk) in Figure 5, the ratio of ions to electrons at the plasma-sheath boundary, $\frac{N_i(0)}{N_e(0)}$, was held constant as indicated in the legend of Figure 5. The ratio of charge densities at the boundary, $\frac{N_i(0)}{N_e(0)}$, includes the ions that are reflected due to the anode potential, while the neutralization parameter, α , only accounts for the ratio of densities of ions to electrons which enter the sheath from the bulk plasma. The charge density ratio at the plasma-sheath boundary does not include any of the ions that are emitted from the wall. The reason for this definition is so that the term can be related back to the charge density of the bulk plasma, and in

all the simulations performed, the charge density of emitted ions was always one to two orders of magnitude smaller than the charge density of particles entering the sheath from the plasma. The total current through the sheath, γ , and the emitted current, λ , were varied and the resulting potential drop across the sheath, ψ_w , and the required neutralization parameter, α , were plotted as a point for each simulation run. Each line connecting points is a constant total current through the sheath, γ , while each individual point is a given emitted current, λ .

Two general trends were noted involving the total current through the sheath, γ , and emitted current, λ . For a constant emitted current, say $\lambda = 0$ (which is no emitted current), as the total current through the sheath, γ , is increased the wall potential increases along with it. Referring back to Eq. (11), the normalized electron current, J_e , is constant, while the ion current to the wall is able to vary based on both the neutralization parameter and wall potential, ψ_w . In order to decrease the flux of ions to the wall, there either needs to be lower ion density in the bulk plasma, or the wall potential must increase to reflect additional ions so that a smaller fraction reach the anode wall. One of the requirements for this plot was to maintain the ratio of ions to electrons at the plasma-sheath boundary. As the wall potential, ψ_w , increases more ions are reflected, and the density of ions at the plasma-sheath boundary increases. In order to maintain the $\frac{N_i(0)}{N_e(0)}$ ratio, the neutralization parameter, α , must be decreased. Because of these factors, as the total current through the sheath, γ , increases, the potential drop across the sheath, ψ_w , increases and the neutralization parameter, α , decreases.

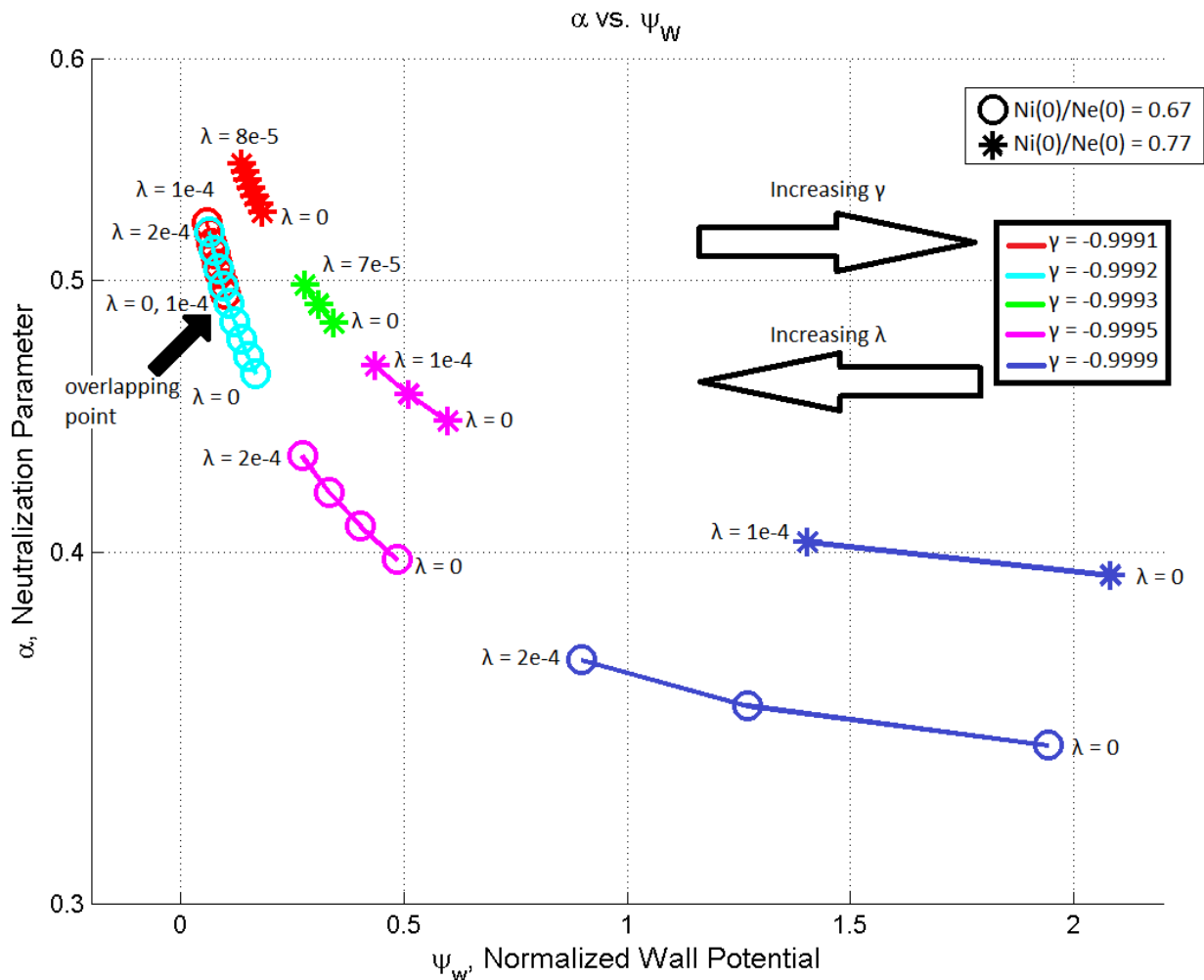


Figure 5. Detail of α - ψ_w parameter space for wall capable of ion emission, plasma-sheath boundary density ratio of 0.67 and 0.77, equal ion and electron temperature and ion mass = 131 amu.

The emitted current, λ , has the opposite effect as the total current through the sheath, γ . In order to maintain a constant total current through the sheath, as the emitted current, λ , increases, so must the ion current to the wall. For the ion current to the wall to increase, the wall potential, ψ_w , must decrease. Because the wall potential decreases,

the neutralization parameter, α , must increase to maintain the same ratio of ions to electrons at the plasma-sheath boundary.

D. Multiple solutions of the parameter space

In Figure 5, there exists an indicated point where two solutions are shown to occupy the same α - ψ_w location, one being on the red-circle curve, the other on the cyan-circle curve. Even though only two solutions are shown at the α - ψ_w point indicated above, there exists a continuous range of solutions that could represent the point indicated. Referring to Eq. (11), for a given plasma condition and neutralization parameter, α , the potential drop across the sheath, ψ_w , is determined only by the difference between the total current through the sheath, γ , and emitted current, λ . However, only a certain range of total current through the sheath, γ , and emitted currents, λ , will provide a monotonically increasing solution. As the emitted current, λ , is increased, the ion density near the wall increases drastically (due to the slow velocity of the emitted ions) and the sheath potential becomes space charged and non-monotonic.

Figure 6 includes the two potential profiles for the two solutions mentioned above that have the same α and ψ_w values. They both have the same difference between the total current through the sheath, γ , and emitted current, λ . Both of the curves in Figure 6 represent the potential profile in the sheath. Both curves have the same bulk plasma temperature and density ratio and the potential drop across the sheath is the same. In Figure 6, the $x=0$ location is the plasma-sheath boundary, and the potential at this location for all profiles is zero. The magnitude of the potential in each curve increases until it reaches the wall, which is where each curve ends. The difference in conditions between the two curves is the total current through the sheath and emission from the wall. The solid curve in Figure 6 represents the case where wall emission is present with $\lambda = 10^{-4}$. One of the differences between the two results is the length of the sheaths. The case with higher current and wall emission has a longer sheath. The major difference between the two cases is the shape of the potential profiles. This difference in potential profile becomes very obvious when the electric field of the two profiles is plotted.

The electric fields of the potential curves in Figure 6 are shown in Figure 7. First, for a given position, the sheath adjacent to the wall emitting ions has a smaller (in magnitude) electric field at every position. Because of this, a longer distance is required to reach the same potential. Another noticeable difference is the electric field near the anode wall. In the case with no ion emission from the wall, the slope of the electric field appears relatively unchanged. The electric field for the case emitting ions from the wall has a distinct reduction in magnitude of electric field. The electric field near the wall decreases because of the high density of the emitted ions near the wall. The ions are emitted at a constant flux, so when they are near the wall they travel at a

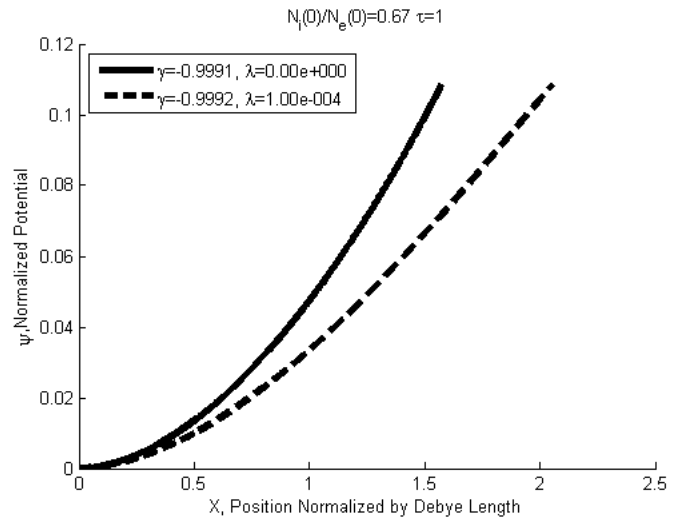


Figure 6. Potential profiles for two solutions with the same α - ψ_w point from Figure 5.

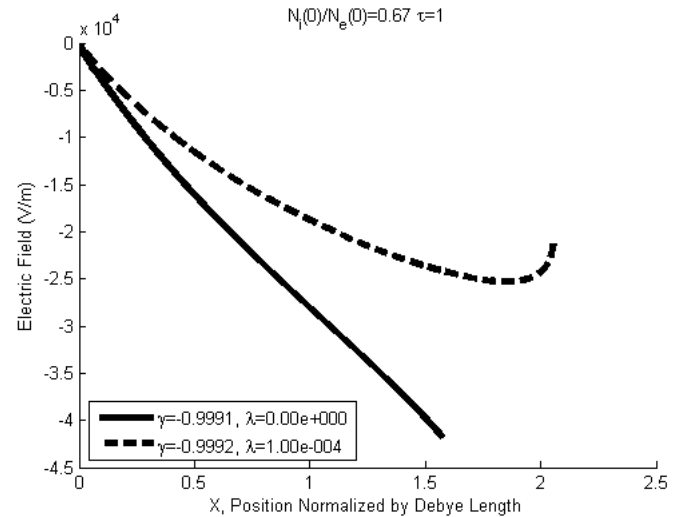


Figure 7. Electric field profiles for two solutions with the same α - ψ_w point from Figure 5.

very low velocity meaning they have a high density. As they travel away from the wall, the velocity increases and the density decreases.

IV. Conclusion

A set of kinetic based equations was developed to model the sheath of a wall that could emit ions into the sheath. The neutralization parameter was examined and it was found that the cathode sheath of a two-electrode discharge contributed to the density of electrons into the anode sheath. This decoupled the neutralization parameter, α , from directly describing the plasma neutrality. The magnitude of the potential drop across the sheath was found to decrease as the emitted current increased. This was found to be due to an increase in the ion current from the plasma. When the magnitude of total current through the sheath was increased, it was found that the wall potential and the length of the sheath both increased.

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