COULOMB FORCE VIRTUAL SPACE STRUCTURES

G. G. Parker, H. Schaub, A. Natarajan and L. B. King

Workshop on Innovative Systems Concepts

Noordwjik, The Netherlands   Feb. 21, 2006

COULOMB FORCE VIRTUAL SPACE STRUCTURES

Gordon Parker[1], Hanspeter Schaub[2], Arun Natarajan[3], and Lyon King[4]

[1]*Michigan Technological University, MEEM Dept., Houghton, MI 49931, USA*
[2]*Virginia Tech, Aerospace and Ocean Engineering Dept., Blacksburg, VA 24061, USA*
[3]*Virginia Tech, Aerospace and Ocean Engineering Dept., Blacksburg, VA 24061, USA*
[4]*Michigan Technological University, MEEM Dept., Houghton, MI 49931, USA*

ABSTRACT

Recently, several uses for intercraft Coulomb forces have been explored. Proposed applications have ranged from creating static formations of many spacecraft to steerable nanosat deployment systems. This paper considers the use of Coulomb forces for creating space structures. Unlike conventional space structures, these "virtual space structures" have no physical connections. Instead, they are held together by maintaining specific charges at their node points. This form of structure is thus readily expandable, can be reconfigured to different shapes, and is easily deployed. Fundamental concepts are covered in conjunction with a control law illustrating the ability to form a structure with conventional stiffness and damping coefficients that can be actively modified.

Key words: Coulomb Force, Formation Flying, Space Structures.

1. INTRODUCTION

The forces generated between two charged bodies in space are sufficiently large that they can be used for a variety of useful purposes (King et al., 2002, 2003). A simplistic example is shown in Figure 1 where the two bodies are perfect spheres. In a vacuum the Coulomb force, f_{12}, is

$$f_{12} = \frac{k_c q_1 q_2}{d^2} \quad (1)$$

where k_c is Coulomb's constant ($8.99 \times 10^9 \frac{Nm^2}{C^2}$), q_1 and q_2 are the charges of the two bodies in Coulombs and d is the distance between their centers in meters.

Charged bodies in a plasma, such as in the vicinity of Earth, have a shielding effect characterized by the Debye length (λ_d). When a node actively charges, the oppositely charged particles in the plasma are attracted to it. As seen by other participating Coulomb force nodes, it is a cloud of particles with a net zero charge, and thus provides no Coulomb force. This Debye shielding phenomenon is modeled as an exponential decrease in Coulomb force with increasing separation distance. The ideal intercraft Coulomb force of Eq. 1 becomes

$$f_{12} = \frac{k_c q_1 q_2}{d^2} e^{-d/L_\lambda} \quad (2)$$

when considering the plasma Debye length shielding. For separation distances of $d > 2\lambda_d$ the Coulomb force effect is negligible.

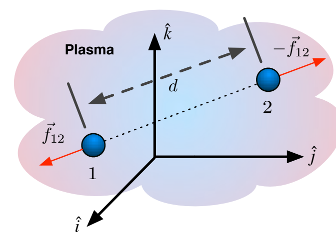

Figure 1. Coulomb forces between two charged spheres.

The Debye length is a function of altitude, but is not a constant quantity due, for example, to changes in solar activity. In general λ_d is quite small at low Earth altitudes (centimeter level). At geostationary altitudes (GEO) it ranges from 10s of meters to 100s of meters. Thus, Coulomb force exploitation for creating virtual structures, or formation flying, is limited to either higher altitude operations, or interplanetary operations where again λ_d is large.

Active control of spacecraft charging has been an important area of study for many years. Differential charging is typically an undesirable phenomenon and can result in arcing between components and electronic failures. The node charging envisioned for a Coulomb virtual structure is not differential, but whole-craft charging. This is a far safer scenario than differential charging, where kilovolt level potentials should be possible.

The space plasma environment has both electrons and H^+ ions. Since the electrons are roughly 2000 times less massive than the ions, a body moving through the near-Earth plasma accumulates more electrons than ions, and thus, tends to charge negative. The active node charging for a Coulomb virtual structure will require kilovolt level charging. Although this seems prohibitively large, the SCATHA spacecraft, launched in 1979, demonstrated net "natural" potentials as high as -14kV as described by Mullen et al. (1986). Using a low power electron emitter, SCATHA was actively charged to 3kV. Recent analysis shows that kilovolt level potential swings can be achieved with very little power and essentially zero propellant (King et al., 2002; Schaub et al., 2003). Achieving additional negative charge, on demand, will require active ion emission. This can be accomplished with existing technology which has proven suitable for spacecraft operations (Riedler and et al., 1997).

The focus of this paper is the use of Coulomb forces for creating space structures. Since these structures do not have physical connections and are held together by the Coulomb forces alone, they will be called "virtual structures." A survey of recent activity in this area is presented. In addition, the analogy to conventional structures is illustrated with a simple two-node structure. Possible avenues for future research directions are also provided.

2. VIRTUAL STRUCTURE CONCEPT

A Coulomb virtual structure is comprised of several nodes whose charges can be actively controlled. Some nodes may have their own propulsion system for providing a net force to the structure's center of mass, while others may have instrumentation providing the scientific functionality of the structure. A typical structure is shown in Figure 2.

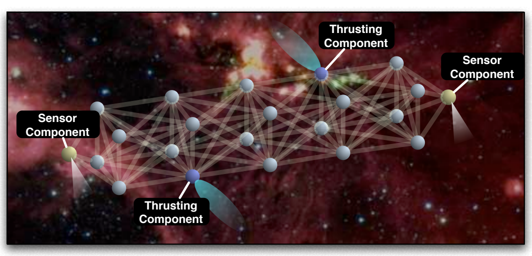

Figure 2. A Coulomb virtual structure illustrating the use of nodes not only for shape control, but also for net structure thrusting and scientific sensors.

The modularity of this type of structure is apparent. Sections could be readily added, as long as the highly coupled charge node interaction effects are considered. Assuming that this system of N charged bodies is orbiting the Earth such that its center of mass has a nominal circular orbit, the Clohessy-Wiltshire (Clohessy and Wiltshire, 1960) or Hill's equations (Hill, 1878) can be used to approximate their Hill-frame relative dynamics. These are shown in Eq. 3 for the N body case.

$$\begin{aligned} m\left(\ddot{x}_i - 2n\dot{y}_i - 3n^2 x_i\right) &= k_c \sum_{j=1}^{N} \frac{x_i - x_j}{d_{ij}^3} q_i q_j e^{-\frac{d_{ij}}{\lambda_d}} \\ m\left(\ddot{y}_i + 2n\dot{x}_i\right) &= k_c \sum_{j=1}^{N} \frac{y_i - y_j}{d_{ij}^3} q_i q_j e^{-\frac{d_{ij}}{\lambda_d}} \\ m\left(\ddot{z}_i + n^2 z_i\right) &= k_c \sum_{j=1}^{N} \frac{z_i - z_j}{d_{ij}^3} q_i q_j e^{-\frac{d_{ij}}{\lambda_d}} \end{aligned} \tag{3}$$

for $i = 1 \dots N$, $j \neq i$ during the summation and $d_{ij} = \|\vec{p}_i - \vec{p}_j\|$. The Hill frame angular velocity is denoted as n and the mass of each node is m. The position of the ith node, relative to the Hill frame origin at the structure's center of mass, is denoted by the components of $\vec{p}_i$ given by x_i, y_i, z_i. Clearly, each charged node interacts with all the other charged nodes in a complex, but well defined way. Simply adding a new section to the structure without taking this into account would result in structural deformation, and eventually the structure's demise due to the orbital dynamic effects on the left side of Eq. 3. It should be noted, however, that for particularly large structures with characteristic lengths greater than $2\lambda_d$, distant nodes will no longer interact due to the Debye shielding effect. Thus, the control problem of actively modulating the node charges to achieve equilibrium could be simplified due to the Debye shielding effect.

3. STATIC VIRTUAL STRUCTURES

One of the earliest attempts at exploiting Coulomb forces was for creating static, crystal-like spacecraft formations. At the heart of this analysis is the solution to Eq. 3 where the speed and acceleration terms are set to zero. The resulting static equilibrium equations are shown in Eq. 4

$$\begin{aligned} -3\tilde{x}_i &= \sum_{j=1}^{N} \frac{\tilde{x}_i - \tilde{x}_j}{d_{ij}^3} \tilde{V}_i \tilde{V}_j e^{-M d_{ij}} \\ 0 &= \sum_{j=1}^{N} \frac{\tilde{y}_i - \tilde{y}_j}{d_{ij}^3} \tilde{V}_i \tilde{V}_j e^{-M d_{ij}} \\ \tilde{z}_i &= \sum_{j=1}^{N} \frac{\tilde{z}_i - \tilde{z}_j}{d_{ij}^3} \tilde{V}_i \tilde{V}_j e^{-M d_{ij}} \end{aligned} \tag{4}$$

where M is the number of Debye lengths considered, and both the positions and charges have been normalized using

$$\begin{aligned} \tilde{x}_i &= \frac{x_i}{M\lambda_d} \\ \tilde{y}_i &= \frac{y_i}{M\lambda_d} \\ \tilde{z}_i &= \frac{z_i}{M\lambda_d} \end{aligned} \tag{5}$$

and

$$\tilde{V}_i = \frac{r_{s/c}}{n\sqrt{m\left(M\lambda_d\right)^3 k_c}} V_i \tag{6}$$

Chong (2002) showed that analytical solutions to the static Coulomb equations of Eq. 4 exist. By enforcing specific shape symmetries she demonstrated structures with up to 6 participating nodes. It should be noted that these were not "free-flying" formations. Each had a central Coulomb node with its own conventional propulsion system. Thus, all the other nodes could react against the central node. Stability was considered after finding equilibrium configurations. It was concluded that none of the shapes was passively stable. Thus they would require an active control strategy to maintain their shape.

This original work illustrated that virtual structures of 10s of meters could be created using Coulomb forces. The next investigation focused on free-flying formations with none of the nodes having a conventional propulsion system described by Berryman and Schaub (2005b) and Berryman and Schaub (2005a). Given a specified number of nodes, a genetic algorithm optimization approach was used to solve for the relative positions and charges such that static equilibrium equations were satisfied. Although there was no prescription as to the desired shape, this illustrated that free-flying virtual structures were possible with up to 9 nodes as shown in Figure 3. Larger clusters are certainly possible, using the optimization approach applied to the 9 node case. It should be noted that Debye length shielding was not considered. Necessary conditions on the shapes were also developed for satisfaction of the equilibrium equations (Schaub et al., 2005). In short, it was required that the structures principle inertia axes be collinear with the Hill frame.

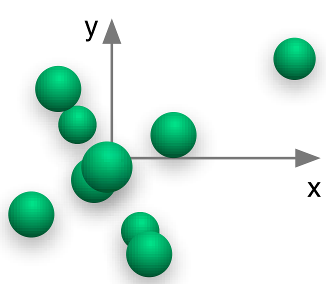

Figure 3. A nine node free-flying virtual structure.

This work was later extended to generating virtual structures that achieved a desired shape using a subset of the nodes as described by Parker et al. (2006a). In general, a Coulomb virtual structure will need more nodes than required of a conventional structure. These extra nodes are needed to generate the proper internodal forces to balance the static equilibrium equations of Eq. 4. For example, an eight node square box structure will not provide enough control degrees of freedom to satisfy the equilibrium equations.

Again, an optimization strategy was used to generate both the node charges and the positions of all the nodes. The specified shape nodes were incorporated as a penalty function in the overall cost function shown in Eq. 7 where $\vec{\tilde{R}}$ are the residual accelerations derived from Eq. 4. The first term of the cost function has two purposes. It simultaneously favors enforcement of the static equilibrium equations while trying to maintain even charges across all the nodes. The second term, with weighting factor w_1, enforces the desired shape function embodied in S. The last term favors satisfaction of the static formation necessary conditions. Debye length shielding was considered in this study where it was shown that specified shapes could be readily constructed. A specific example is shown in Figure 4 where the goal was to maintain three of the five nodes in a triangle as viewed by an observer on Earth located directly below the structure.

$$J = \left(\frac{\max\left|\tilde{V}_i\right|}{\min\left|\tilde{V}_i\right|}\right)\sum_{i=1}^{N}\left|\vec{\tilde{R}}_i\right| + w_1 S(\vec{\tilde{p}}_i, \vec{L_p}) + w_2\Big\{\,|I_{xy}| + |I_{xz}| + |I_{yz}|\,\Big\} \tag{7}$$

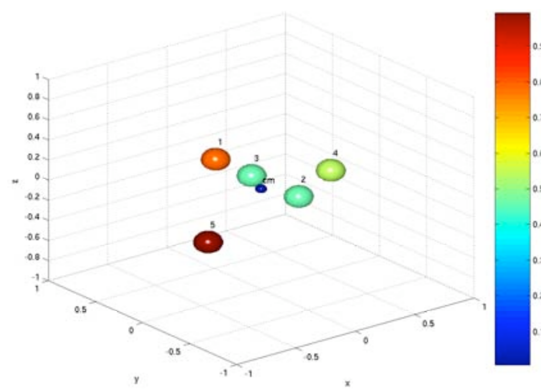

Figure 4. A five node structure with 3 nodes forming an equilateral triangle.

The color bar illustrates the normalized node voltages, scaled to that of the maximum node voltage. For an equilateral side length of 12 meters, a λ_d of 50 meters, and a node radius of 0.5 meters, the maximum node voltage was 16 kilovolts.

4. VIRTUAL STRUCTURE ANALOGY TO CONVENTIONAL SPACE STRUCTURES

From the previous analyses two things are evident. First, practically sized virtual structures are feasible. Second, some form of closed-loop control will be needed to maintain the structure's shape. The difficulty of implementing a control strategy, due to the dynamic coupling between nodes, has already been mentioned. A charge cycling approach has been suggested for effectively decoupling the node interaction to all but two participating nodes at any one time (Parker et al., 2006b). In addition, the ability to create (and modify) conventional structure vibration characteristics in Coulomb virtual structure using closed-loop control is possible using the method described by Natarajan and Schaub (2005).

Consider a simple, two node structure such that each node is nominally on the Hill frame x axis, that is, along a line directed radially out from the Earth. Taking as degrees-of-freedom the separation distance between the nodes (L), the in plane rotation of the structure (ψ) and the out of plane rotation (θ), the Hill dynamic equations can be represented as shown in Eq. 8 where δ denotes a small perturbation from the nominal configuration and $L_{\rm ref}$ is the desired separation distance that can, in general, be a time-varying quantity.

$$
\begin{aligned}
&\ddot{\theta} + \frac{2\dot{L}_{\rm ref}}{L_{\rm ref}}\dot{\theta} + 4n^2\theta = 0 \\
&\ddot{\psi} + \frac{2\dot{L}_{\rm ref}}{L_{\rm ref}}\dot{\psi} + \frac{2n}{L_{\rm ref}}\delta\dot{L} - \frac{2\dot{L}_{\rm ref}}{L_{\rm ref}^2}n\delta L + \frac{2\dot{L}_{\rm ref}}{L_{\rm ref}}n + \\
&\quad 3n^2\psi = 0 \\
&\delta\ddot{L} + \ddot{L}_{\rm ref} - 2nL_{\rm ref}\dot{\psi} - 9n^2\delta L - \\
&\quad \frac{k_c}{m_1}\delta Q \frac{1}{L_{\rm ref}^2}\frac{m_1+m_2}{m_2} = 0
\end{aligned}
\tag{8}
$$

The resulting set of linearized equations is time-varying if the reference separation distance, $L_{\rm ref}$ is specified to change. Postulating the control law of Eq. 9

$$\delta Q = \frac{m_1 m_2 L_{\rm ref}^2}{(m_1+m_2)\,k_c}\left(-C_1\delta L - C_2\delta\dot{L} + 2nL_{\rm ref}\dot{\psi}\right) \tag{9}$$

where the constants C_1 and C_2 are the position and velocity feedback gains, results in a closed-loop virtual structure which has a specifiable axial stiffness and damping. Specifically, the closed-loop δL equation becomes

$$\delta\ddot{L} + C_2\delta\dot{L}\left(C_1 - 9n^2\right)\delta L = 0 \tag{10}$$

The structure's axial mode natural frequency and damping ratio are

$$
\begin{aligned}
\omega_n &= \sqrt{C_1 - 9n^2} \\
\zeta &= \frac{C_2}{2\sqrt{C_1 - 9n^2}}
\end{aligned}
\tag{11}
$$

and can be prescribed independently by selecting the control gains C_1 and C_2 appropriately.

This analogy is only meaningful if the entire system of Eq. 8 is stable. If $L_{\rm ref}$ is constant, then the stability bounds on C_1 and C_2 are readily computed by analyzing the characteristic equation of Eq. 8 and shows that $C_1 > 9n^2$ and $C_2 > 0$ are required. Stability of the time-varying system requires bounds on $\dot{L}_{\rm ref}$ and has been considered by Natarajan and Schaub (2005).

5. FUTURE RESEARCH DIRECTIONS

In general the exploitation of Coulomb forces for spacecraft missions is an open area of research. Fundamental and experimental work is needed in the area of charge sensing and modulation. This would benefit the full spectrum of applications, including Coulomb virtual structures. Control strategies are needed to accommodate the complexity associated with the coupled nonlinear dynamics of an N node Coulomb virtual structure. A direct approach would be to devise a coupled nonlinear strategy that yields desired positioning performance while guaranteeing stability. Indirect methods may also prove effective where subsets of the structure are decoupled. The idea being that the solution to several reduced order problems would likely be more tractable than designing a control system for one high order system.

Active reconfiguration is a potential benefit of Coulomb virtual structures that has not been fully investigated. While conceptually easy, this may prove to be quite challenging. It is likely that exploitation of orbital dynamics will by beneficial. This may require development of trajectory design strategies that guarantee the controllability of the structure throughout its metamorphosis between shapes.

Consideration of the analogous structural properties of a Coulomb virtual structure may facilitate the use of existing space structure control techniques. This would require significant work beyond that described above for the simple two-node system. If successful, it may be possible to tailor the structure's natural frequencies and mode shapes to suit changing rigidity requirements. For example, while vibration sensitive scientific instruments are active, its host subsystem may be decoupled from the rest of the structure, achieving true isolation.

REFERENCES

Berryman, J. and Schaub, H. (2005a). Analytical charge analysis for 2- and 3-craft coulomb formations. Lake Tahoe. Paper No. 05-278.

Berryman, J. and Schaub, H. (2005b). Static equilibrium configurations in geo coulomb spacecraft formations. In *AAS/AIAA Space Flight Mechanics Meeting*, Copper Mountain, Colorado. Paper No. AAS 05-104.

Chong, J.-H. (2002). Dynamic behavior of spacecraft formation flying using coulomb forces. Master's thesis, Michigan Technological University.

Clohessy, W. H. and Wiltshire, R. S. (1960). Terminal guidance system for satellite rendezvous. *Journal of the Aerospace Sciences*, 27(9):653–658.

Hill, G. W. (1878). Researches in the lunar theory. *American Journal of Mathematics*, 1(1):5–26.

King, L. B., Parker, G. G., Deshmukh, S., and Chong, J.-H. (2002). Spacecraft formation-flying using inter-vehicle coulomb forces. Technical report, NASA/NIAC. `http://www.niac.usra.edu`.

King, L. B., Parker, G. G., Deshmukh, S., and Chong, J.-H. (2003). Study of interspacecraft coulomb forces and implications for formation flying. *AIAA Journal of Propulsion and Power*, 19(3):497–505.

Mullen, E. G., Gussenhoven, M. S., and Hardy, D. A. (1986). Scatha survey of high-voltage spacecraft charging in sunlight. *Journal of the Geophysical Sciences*, 91:1074–1090.

Natarajan, A. and Schaub, H. (2005). Linear dynamics and stability analysis of a coulomb tether formation. In *15th AAS/AIAA Space Flight Mechanics Meeting*, Copper Mountain, Colorado. Paper No. AAS 05-204.

Parker, G., King, L., and Schaub, H. (2006a). Charge determination for specified shape coulomb force virtual structures. In *47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Newport, RI.

Parker, G., King, L., and Schaub, H. (2006b). Steered spacecraft deployment using interspacecraft coulomb forces. In *American Control Conference*, Minneapolis, MN.

Riedler, W. and et al. (1997). Active spacecraft potential control. *Space Science Reviews*, 79:271–302.

Schaub, H., Hall, C., and Berryman, J. (2005). Necessary conditions for circularly-restricted static coulomb formations. In *AAS Malcolm D. Shuster Astronatuics Symposium*, Buffalo, NY. Paper No. AAS-05-472.

Schaub, H., Parker, G. G., and King, L. B. (2003). Challenges and prospect of coulomb formations. In *AAS John L. Junkins Astrodynamics Symposium*, College Station, TX. Paper No. AAS-03-278.