Electron Dynamics of a Non-neutral Electron Plasma in the Defining Fields of a Hall-effect Thruster

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Abstract: An electron trapping apparatus was constructed in order to study electron dynamics in the defining fields of a Hall-effect thruster. The approach presented here decouples the cross-field mobility from plasma effects by conducting measurements on a pure electron plasma in a pristine environment. A purely radial magnetic field is applied with a crossed, independently-controlled, axial electric field in order to evaluate electron trajectories and cross-field mobility in response to field strength and collision frequency. Without neutral plasma effects, such as density fluctuations and turbulence, mobility is presumed to follow the classical mobility model. In the present research, measurement techniques are investigated and verified against the classical model. Preliminary findings suggest that that the apparatus and techniques used will be valid for mobility studies in more complex field environments.

Nomenclature

A_a	=	anode surface area
A_p	=	probe surface area
B_r	=	radial magnetic field
ε_0	=	permittivity of free space
E_z	=	axial electric field
I_d	=	mobility drift current
I_p	=	probe current
$J_{e\theta}$	=	Hall current density
J_{ez}, J_a	=	cross-field electron current density, current density at the anode
J_p	=	current density at the probe
m_e	=	electron mass
μ_{ez}	=	cross-field electron mobility
n_e	=	electron number density
ν_{ne}	=	electron-neutral collision frequency
q	=	electron charge
$u_{e\theta}$	=	azimuthal electron velocity
u_{ez}	=	cross-field electron velocity
v_e	=	electron perpendicular velocity
Φ	=	electric potential
ω_{ce}	=	electron cyclotron frequency
Ω_H	=	electron Hall parameter

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I. Introduction

THE defining characteristic of Hall thrusters is the crossed axial electric and radial magnetic fields. The criteria of the E- and B-fields is such that the electron gyro radius is small compared with apparatus dimensions while the gyro radius and mean free path for ions are larger than apparatus dimensions. The purpose of the radial magnetic field is to impede electron flow toward the anode, which induces the confining azimuthal **E**x**B** electron drift. This impeded motion creates the electrostatic field necessary to accelerate large propellant ions, whose trajectories are not significantly affected by the magnetic field.

Ideally, electrons would be indefinitely confined in the azimuthal **E**x**B** drift, as transverse electron mobility clearly has a negative effect on thruster efficiency. However, in past experiments the cross-field electron mobility was found to be much larger than the classical collisional diffusion model. This "anomalous" mobility was first observed by Janes and Lowder[1] and later supported by Meezan et al[2], who observed mobility up to 1,000 times greater than predicted by classical theory. This departure from classical mobility has been shown where the magnetic field strength is high and intense plasma fluctuations reportedly exist. These studies, among others, hypothesized that electric field instabilities produced by fluctuations in the plasma density may be a mechanism responsible for the anomalous mobility.

The approach used in this investigation removes such plasma fluctuations by examining the electron dynamics of a pure electron plasma in an Hall thruster's defining fields with an independently-controlled electric field. Thus, mobility studies are performed in a pristine environment disconnected from the coupling ion and neutral plasma effects that usually control *and* are controlled by the field environment. While this approach has not been documented in Hall thruster studies, this method has proven to be useful for numerous types of charged particle transport studies,[3,4] examples being the Electron Diffusion Gauge (EDG) experiments of Chao and Davidson,[5] the measurement of neo-classic mobility by Robertson,[6,7,8] and numerous other related work involving positrons and ions.[9]

II. Mobility Concepts

Although electron mobility studies in a Hall thruster's defining fields are well documented for a neutral or quasi-neutral plasma, certain considerations must be made when translating these concepts to a non-neutral electron plasma. In a Hall thruster the magnitude of the B-field is adjusted such that the electron Larmor radius is much less than the thruster size while the ion Larmor radius is much greater. The net effect is that the electron trajectories are controlled by both the magnetic and electric fields, while the ion motion is affected only by the electric field and the resulting motions can be uncoupled. From this, it follows that the predominant electron motion is the azimuthal ExB drift, since electrons are allowed to gyrate around B-field lines within the trap dimensions. The only electron motion across B-field lines is due to electron-neutral or electron-wall collisions, which allow an electron to "jump" to a new field line closer to the anode. In purely axial electric and radial magnetic fields, the electron momentum equation gives the electron current density in the axial direction,

$$J_{ez} = qn_e u_{ez} = qn_e \left(\frac{E_z}{B_r}\right)\frac{\nu_{ne}}{\omega_{ce}} \quad . \tag{1}$$

Here the cross field electron mobility is defined as a constant of proportionality between cross-field velocity of electrons and the electric field:

$$\mu_{ez} \equiv \frac{u_{ez}}{E_z} = \left(\frac{1}{B_r}\right)\frac{\nu_{ne}}{\omega_{ce}} = \frac{m_e \nu_{ne}}{qB^2} \quad . \tag{2}$$

Considering electron motion in these fields gives rise to the electron Hall parameter,

$$\Omega_H = \frac{\omega_{ce}}{\nu_{ne}}, \tag{3}$$

which must be sufficiently large for classical mobility studies to be valid in a Hall thruster.

In a typical Hall thruster the impeded electron motion creates the electrostatic field necessary to accelerate propellant ions. However, the approach used in this investigation reproduces the Hall thruster's defining fields with an externally-controlled electric field in order to examine the dynamics of a pure electron plasma. The purpose of this approach is to provide a pristine evaluation of individual electron trajectories, decoupled from plasma effects. The fundamental difference between these non-neutral plasma experiments and the quasi-neutral Hall thruster plasma is the existence of negative space charge created by mutual repulsion of the electrons in the absence of screening ions. The mutual repulsion will induce a self-field that will drive the electrons to the walls of the trap; however, the magnitude of the space-charge field can be made negligible by adjusting the electron density.

The magnitude of the induced negative space charge is found from a solution of the Poisson equation $\nabla^2\Phi = qn_e/\varepsilon_0$, where Φ is the electric potential. Since the experiments presented here are sensitive primarily to the radial space-charge field (the axial space-charge field will be much less than the applied E-field and will be a negligible perturbation), the experimental geometry can be approximated as a cylindrical annulus infinite in the z-dimension with inner radius r_{in} and outer radius r_{out}. If the space between the cylinders is filled uniformly with electrons at density n_e=*constant*, then the solution to the Poisson equation for the space-charge potential relative to the trap walls is

$$\frac{\Phi(r)}{n_e} = \frac{e}{4\varepsilon_0}\left[r_{out}^2 - r^2 + \left(r_{out}^2 - r_{in}^2\right)\frac{\ln(r_{out}/r)}{\ln(r_{in}/r_{out})}\right] \tag{4}$$

The maximum potential will occur midway between r_{in} and r_{out} where, for the physical scale proposed here, $\Phi_{max}/n_e \sim 1\text{x}10^{-12}$ Vm^3. Thus, if the electron density is limited to $n_e < 5\text{x}10^{10}$ m^{-3} (very typical for electron non-neutral particle traps[5]), then the space-charge potential at chamber mid-radius will be less than 50 mV below wall potential. The moderate radial electric field induced by this 50-mV difference will not significantly alter the electron trajectories. For typical field configurations to be investigated with this trap, namely $B_r \sim 100$ G and $E_z \sim 1\text{x}10^4$ V/m, an electron cloud with density ~$5\text{x}10^{10}$ m^{-3} creates a measurable 20-nA Hall current on an electrostatic probe area during in-situ measurements of n_e. During the course of experiments electron production was monitored and controlled to keep the electron density well within this range.

III. Description of Methods

The cross field mobility is evaluated experimentally using a steady state method. The Hall current density is given by

$$J_{e\theta} = qn_e u_{e\theta}. \tag{5}$$

Using a probe measurement of the Hall current density, an estimate of n_e can be determined, which when combined with electron current incident on the anode, allows the mobility to be directly evaluated. This is demonstrated by realizing that the transverse mobility, μ, is related to the electron density, n_e, the axial current flux, J_{ez}, and the axial E-field, E_z, by $J_{ez}=en_e\mu E_z$. Thus, if the anode collects electron current I_d over an area A_{anode}, the mobility can be directly calculated from

$$\mu = \frac{I_d}{qn_e A_{anode} E_z}. \tag{6}$$

A probe measurement can be used assuming that the Hall parameter is sufficiently high so that the dominant electron motion is the azimuthal **E**x**B** drift. With this assumption in place, the Hall current density can be measured where $I_p = qn_e u_{e\theta} A_{probe}$. Thus, equation (6) simply becomes a ratio of the current density at the anode to the current density measured by the probe scaling with B_r^{-1}:

$$\mu = \left(\frac{1}{B_r}\right)\frac{I_d A_p}{I_p A_a} = \left(\frac{1}{B_r}\right)\frac{J_a}{J_p}. \tag{7}$$

This method has the benefit that the mobility is indicated by the probe current density ratio, rather than the absolute values of each current. This uncouples the effect of varying electron emission or pair production rate on the measured mobility. This probe measurement is also used to determine an estimate of n_e in order to ensure all experiments are conducted within the space charge limitation described previously.

IV. Description of Apparatus

A. Electron Trapping Apparatus

An electron trapping device shown in Fig. 1(a) was constructed to reproduce the defining characteristics of a Hall thruster's accelerating region. The electric field in the region of interest is known independently and formed through parallel plates in vacuum, rather than via a self-consistent plasma. In place of a typical Hall thruster's cusped magnetic field, a wide pole magnetic field is constructed where the field decreases radially but has only slight axial variations as shown in Fig. 1(b). The insulating walls of the trap are constructed of alumina silicate ceramic. In this configuration the magnetic and electric fields are essentially orthogonal inducing the confining azimuthal **E**x**B** drift in typical Hall thrusters.

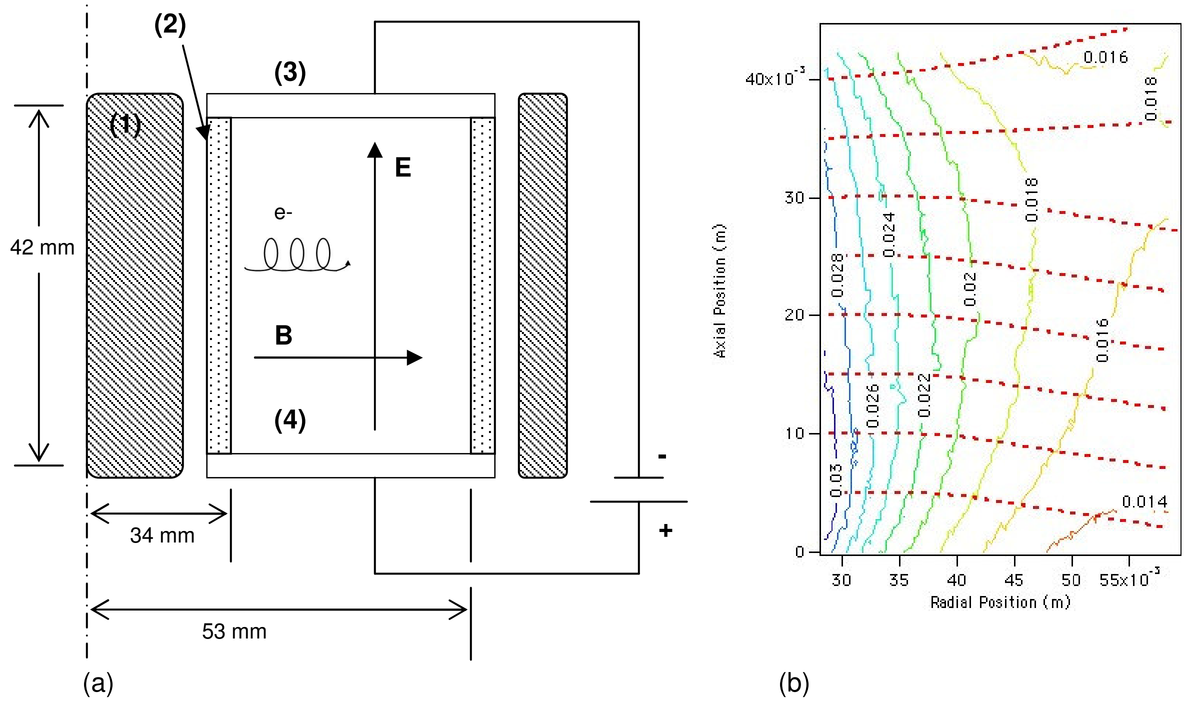

Figure 1. Schematic of Trapping Apparatus (a) and Simulated B-field (b) *The main features of the trapping apparatus are shown; (1) magnetic pole; (2) ceramic insulator wall; (3) plate acting as the cathode; (4) plate acting as anode; The simulated B-field is shown to the right where the solid lines represent lines of constant B-field and the dashed lines represent magnetic field lines.*

In the steady state analysis, the trap is loaded with electrons by passing a ~75 eV ionizing electron beam through a radial slot in the trap (Fig. 3). While the high-energy beam electrons will pass through unaffected, electron-ion pairs will be created within the trap by collisions with background gas. The unmagnetized ions immediately re-combine at the cathode, while the low-energy electrons are trapped in azimuthal orbits via the applied fields. The trap assumes a steady state where the rate of electrons being collected at the anode due to the mobility drift is equal to the rate of electrons being formed inside the trap. A 0.127-mm-diameter, 6-mm long tungsten probe is positioned 180 degrees opposite the electron gun, such that the probe does not collect current due to the high energy primaries but collects only low-energy, secondary electrons confined within the trap. The probe is biased to the local potential of the trap as to not perturb the electric field of the trap and positioned 2 mm closer to the anode in order to collect electrons that have been produced by the electron beam and confined within the trap. An electrical schematic of the entire setup is shown in Fig. 3.

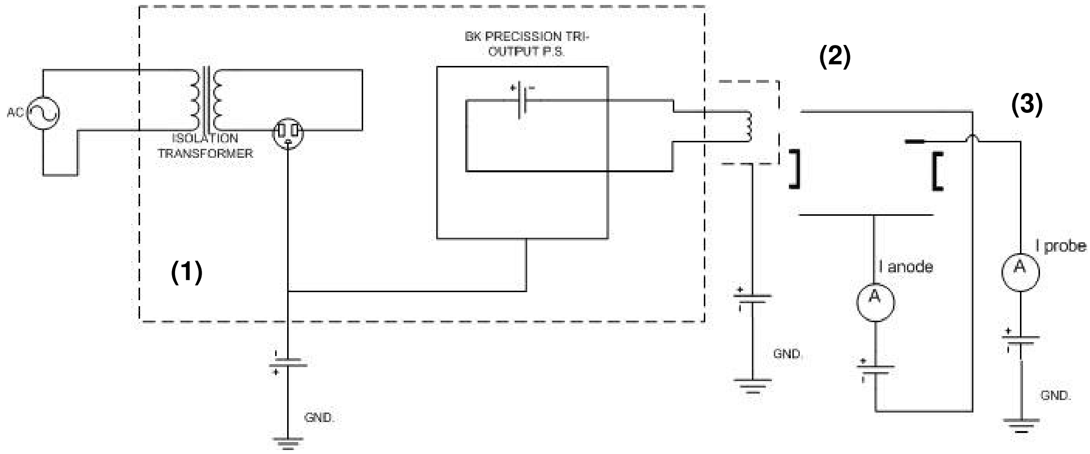

Figure 3. Electrical Schematic for Steady State Method. *The main features of the electrical setup for the transient method are shown; (1) filament setup and bias; (2) trapping apparatus; (3) electrostatic probe setup.*

B. Testing Facilities

All testing was performed in the Ion Space Propulsion Lab (Isp Lab) at Michigan Technological University. The testing facility consists of a 2m diameter, 4m long cylindrical vacuum chamber. Rough pumping is accomplished through a two-stage mechanical pump, capable of 400cfm. High vacuum is achieved through the use of three turbomolecular pumps with a combined throughput of 6,000 liters per second providing a base pressure below 10^{-6} Torr. Xenon gas was introduced to increase the base pressure from 10^{-6} up to 10^{-4} Torr.

V. Results and Discussion

The current ratio, J_a/J_p, as it varies with neutral gas pressure is shown in Fig. 4. The E- and B-fields were held constant at $1\text{x}10^4$ V/m and ~100 G, respectively, while neutral xenon gas was introduced to raise the background pressure from ~10^{-6} to ~10^{-4} Torr. Classical mobility scales linearly with collision frequency and thus with background pressure as well. The results in Fig. 4 indicate that classical mobility is indeed observed in this investigation. However, there is a large uncertainty at high pressures that is introduced due to the probe current measurement. At low values of probe current, the uncertainty in the measurement is on the same order of the currents measured; the predicted uncertainty is shown with error bars in Fig 4.

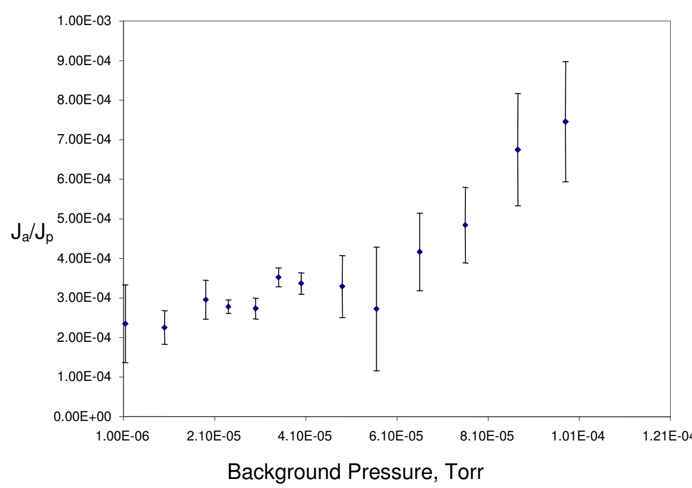

Figure 4. Current density ratio J_a/J_p as a function of background pressure.

The current ratio, J_a/J_p, as it varies with B^{-1} is presented in Fig. 5. The E-field was held constant at $1\text{x}10^4$ V/m and the background pressure was maintained at $1.3\text{x}10^{-6}$ Torr. Since classical mobility scales with B^{-2}, as demonstrated by Eq. (2), it is expected from Eq. (7) that the current ratio would scale linearly with B^{-1}. This scaling is apparent in Fig. 5 within a reasonable certainty level.

Uncertainty is introduced at low levels of B where conditions may give inaccurate probe readings and may be in violation the Hall parameter criteria. In the condition where the B-field is the weakest (~30G) the electron gyro radius of a 2eV electron is ~2 mm, given by $r_L=m_e v_e/qB_r$, which is an order of magnitude larger than our probe diameter and approaching characteristic dimensions of the trap. Thus, measurements taken under these conditions have a large amount of uncertainty as shown with error bars in Fig. 5.

The effect of electron emission current, pair-production and consequently electron density n_e on the current ratio is shown in Fig. The electron density was varied in a range below the space charge limitation by controlling both the heating current and the gate voltage, while holding all other parameters constant. A representative case is shown where the B field is ~150G, the electric field is held constant at $1x10^4$ V/m and background pressure is maintained at $1.3x10^{-6}$ Torr. Because the desired parameter is a ratio of J_a/J_p which should only depend on mobility if the space charge is negligible, this magnitude should not significantly affect the experimentally determined mobility. Fig. shows that over an increase in electron density by a factor of 5 there was no significant change in the parameter J_a/J_p. If the trap conditions exhibit an electron density well below the space-charge limitation, J_a/J_p remains fairly constant with emission current

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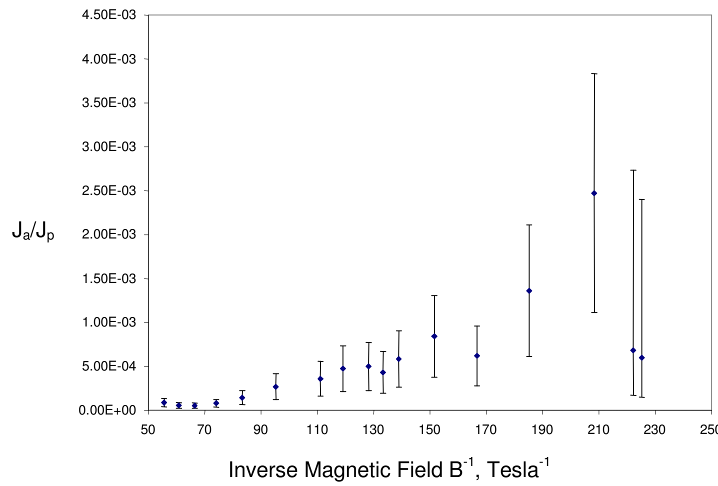

Figure 5. Current density ratio J_a/J_p as a function of B^{-1}.

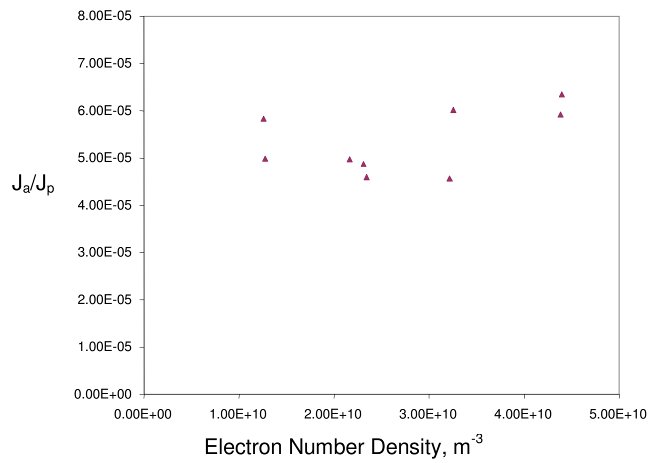

Figure 6. Current density ratio J_a/J_p as a function of electron number density.

VI. Conclusions

This paper presents a preliminary description of an apparatus that can be used to investigate electron mobility in Hall thrusters. The approach presented decouples the cross-field mobility from complicating plasma turbulence and density fluctuations by conducting measurements on a pure electron plasma in a pristine environment. This method for studying electron mobility, using a non-neutral plasma, is new to Hall thruster research. The first stage of this research was to measure mobility in simplified fields and validate it against the classical mobility model. The preliminary findings suggest that this method is suitable for Hall thruster research and will continue to be valid in more complex field environments. This ability to decouple electron motion from plasma effects and control the electric field externally gives rise to a wealth of experiments involving optimized magnetic fields or externally applied electrostatic perturbations.

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